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On Population Dynamics: A Mathematical Model For Two Species Competition In Finite Space

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ABSTRACT

A population has a birth rate and a death rate. Assuming that the increase in the density of a species population causes the birth rate to decrease and the death rate to increase, Pear[1] developed the following differential equation for population growth.

$$p' = rp - bp^2$$

where b reflects the degree to which the density decreases the rate r of increase of the species population p at any time. This matematical model assumes that the population consists of one species only - a rather isolated phenomenon in nature. A number of species are competing for a limited ecological environmental resource and the population density of one species may affect the rate of inrease of the other. In this paper, an attempt is made to study a mathematical model for two species competition for a common resoure in a finite space.

Key Words: ***Population, limited resource, competition.***

INTRODUCTION

Theoriginal work in population dynamics was done due to Malthus[2] by developing a differential equation of order one for population growth of a species as follows :-

$$p' = rp \qquad (1)$$

which leads the exponential growth equation

$$p = po \exp (rt)$$

where p(t) denotes the population at some time t, po is the population size at some arbitrarily set time t = 0 and r is the rate of population increase, that is difference of birth rate over death rate. Malthus law of exponential growth is regarded first principle of population dynamics (Turchin[3]). Malthus is often considered a forerunner of social Darwinism (Shermer[4]).

This exponential growth equation implies that population density $p \to \infty$ as $t \to \infty$. But the space being finite, population can not be infinite. As the population increases due to over crowding and limitations of resources, the birth rate decreases and the death rate increases with the population size p. Pear[1] assumed that by crowding, the rate r is reduced by some proportion b > 0 of the population p and developed the following logistic equation for the rate of population growth of a species.

$$p' = rp - bp^2 \qquad (2)$$

All these mathematical models assumed that the population consists of one species only - a rather isolated phenomenon in nature. A number of species are competing for a limited ecological environmental resource and the population density of one species may affect the rate of inrease of the other. The present paper deals with two species competing for utilization of limited common resource. We reach planetary limits is an open question(Burger[5]). An attempt has been made to show that under a particular set of environmental conditions one species or the other becomes extinet in the competition.

FORMULATION OF MODEL AND ITS SOLUTION

If b_{11} and b_{22} represent the effect of the density of a species on its own rate of increase, and b_{12} and b_{21} the effect of the other species on a species rate of inrease, two species competition logistic growth model is given as follows (Kapur[6]) -

$$p_1' = (r_1 - b_{11}p_1 - b_{12}p_2)\, p_1 \qquad (3a)$$
$$p_2' = (r_2 - b_{21}\, p_1 - b_{22}\, p_2)\, p_2 \qquad (3b)$$

where p_1 denotes the population density of species 1 and p_2 the population density of species 2; and r_1 and r_2 denote corresponding rates of increase of the populations.

Since there is no explicit solution of these non linear differential equations in general, the model can not be fitted to real data easily (Vandermeer[7]).

In the present paper, an attempt has been made to solve these equations under the assumption of a particular set of environmental conditions that the effect of the other species on a species rate of increase is the same as the effect of the density of a species on its own rate of increase, that is, we assume that

$b_{12} = b_{11}$ and $b_{21} = b_{22}$ where $b_{12} > 0$, $b_{21} > 0$ (4)

In case $b_{12} = 0$, $b_{21} = 0$ equations (3a) and (3b) reduce to simple logistic equation (2).

Under the assumptions (4), equation (3a) and (3b) reduce to

$$p_1' = (r_1 - b_{11} P)\, p_1 \qquad (5a)$$

$$p_2' = (r_2 - b_{22} P)\, p_2 \qquad (5b)$$

where $P = p_1 + p_2$

Divide equation (5a) and (5b) by $1/b_{11}p_1$ and $1/b_{22}\, p_2$ respectively and subtract, then we get

$$(1/b_{11}\, p_1)p_1' - (1/b_{22}\, p_2)p_2' = r_1/b_{11} - r_2/b_{22}$$

Where $b_{11} > 0$ and $b_{22} > 0$

Then the integration of the differential equations leads to

$$p_1^{1/b11} / p_2^{1/b22} = A \exp (c_{12}\, t)$$

Where $c_{12} = (r_1\, b_{22} - r_2\, b_{11}) / b_{11}\, b_{22}$

where A is constant of integration. Its value is given by $(p_1)_0^{1/b11} / (p_2)_0^{1/b22} = A$

Where $(p_1)_0$ and $(p_2)_0$ are the values of p_1 and p_2 at time $t = 0$, that is, at the beginning of observation period.

CONCLUSION

From equation (6) the following result is obtained :-

(i) If $c_{12} = 0$, that is, $r_1\, b_{22} = r_2 b_{11}$ then $p_1^{1/b11} = A\, p_2^{1/b22}$
Thus we infer that the ratio of populations of two species remains constant with the passage of time.

(ii) If $c_{12} > 0$, that is $r_1\, b_{22} > r_2\, b_{11}$; and $t \rightarrow \infty$ then $p_1^{1/b11}/p_2^{1/b22} = \infty$
That is $p_2 \rightarrow 0$
Thus we may infer that species 2 becomes extinct.

(iii) If $c_{12} < 0$, that is $r_1\, b_{22} < r_2\, b_{11}$; and $t \rightarrow \infty$ then $p_1^{1/b11}/p_2^{1/b22} = 0$
That is, $p_1 \rightarrow 0$,
Thus we may infer that species 1 becomes extinet.

Hence it was found that in two species competition one species or the other becomes extinct and the extinction depends on the set of environmental conditons. In many experimental studies made by Park[8] under different sets of conditions the extinction of one species or the other has been observed.

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